Coulomb Drag in Systems with Tunneling Bridges

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We study the Coulomb drag effect in double layer electronic systems with local tunneling links. The possibility of tunneling between the layers leads to a pronounced exchange contribution to the transconductance, which is negative and non–vanishing at zero temperature. The diffusive motion of the electrons inside each layer in interplay with the electron–electron interaction cause a distinguishable singular temperature dependence of the transconductance at low temperatures.

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Coulomb drag in spatially separated double–layer electronic systems has received a great deal of attention recently, both experimentally [1–5] and theoretically [6–12]. A setup for studying the drag effect consists of applying current to one of the two layers, and measuring the induced voltage in the second layer. The ratio between the two, known also as the *transresistance*, ρ_D, is the main characteristic of the drag effect. Theoretically, it is more convenient to study the *transconductance*, σ_D, which is related to transresistance as (for $\sigma_D \ll \sigma$)

$$\rho_D = -\sigma_D/\sigma^2, \qquad (1)$$

where $\sigma = e^2 D\nu$ is the conductance of a single isolated layer (hereafter we discuss the case of identical layers); D is the diffusion constant, and ν is the density of states per unit area. Most of the previous studies were concentrated on systems, in which the coupling between the layers is only due to the interlayer Coulomb interaction. In this case density fluctuations of the electron liquid in the first layer induce fluctuations in the second one, which in turn lead to an induced current [6–10]. Since it involves classical thermal fluctuations, interacting in a Hartree way, the drag effect vanishes at zero temperature. Moreover, the sign of the effect does not depend on the nature of the interaction (repulsion or attraction) and has to do only with the sign of the charge carriers in the two layers. For example, for carriers of the same type, the drag transconductance is strictly *positive*, i.e., the induced current flows in the direction of the applied one.

In this paper we address a qualitatively different mechanism which leads to a transconductance. It takes place in the presence of pointlike shortages (or bridges) — points where electrons may tunnel between the two layers. Such bridges are often present in metallic double–layer systems. As we show below the interplay between tunneling and the Coulomb interactions lead to a transconductance which is non-vanishing at zero temperature and *negative* (for carriers of the same charge). That is, the induced current flows in the direction opposite to the driving one. The origin of the effect is in the *exchange* interaction between the electrons of different layers, which is possible due to the wavefunctions' overlap at the bridges. To illustrate the physics of the effect consider a wavepacket propagating in the first layer in the direction of an applied current. Once it reaches a bridge, a part of the wavepacket tunnels into the second layer, while the remaining part continues to move in the initial direction. Without interlayer electron–electron (e–e) interaction, the wavefront propagating in the second layer is spherically symmetric, thus no net current is induced. (Indeed, after tunneling through a point contact the electron completely "forgets" the initial direction of its momentum.) In the presence of interaction the wavefront moves in a direction that minimized its interaction with the initial wavepacket. Thus, for a repulsive interaction the electrons that tunneled move in a direction opposite to that before tunneling. This leads to a *negative* transconductance. As we shall see below such a lowest order (in tunneling) mechanism is dominant only at not too small temperatures. At lower temperatures the leading mechanism involves coherent tunneling to the second layer and back to the first one accompanied by intralayer Coulomb interactions. The exchange contribution, unlike the Hartree term, is not proportional to a small electron–hole asymmetry factor [6–10], therefore, the former's absolute value may well overcome the latter's even for a small tunneling rate. In the case of diffusively propagating wavepackets the exchange transconductance has a peculiar temperature dependence, which is described below.

The tunneling rate between the two layers may be described by a mean intralayer lifetime, τ_{12}, which is related to the *interlayer* tunneling conductance per unit area, $\sigma_\perp$, as

$$\sigma_\perp = e^2 \nu / \tau_{12}. \qquad (2)$$

We shall assume that the tunneling is weak, i.e., $\tau_{12} \gg \tau$, where τ is the mean elastic scattering time within each layer. Three energy scales determine the temperature dependence of the transconductance: $\hbar/\tau_{12} < (\kappa d)\hbar/\tau_{12} < \hbar/\tau$ [13], where $\kappa = 2\pi e^2 \nu$ is the inverse Thomas–Fermi screening radius, and d is the distance between the layers.

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At high temperatures, $\hbar/\tau < T$, the motion of the wavepackets is ballistic and the exchange contribution to the transconductance is negative and temperature independent [its value is discussed below, see Eq. (18)]. At $T < \hbar/\tau$ the diffusive character of the electron motion should be taken into account. Up to the lowest order in the tunneling amplitude we find the following temperature dependent contribution to the transconductance

$$\sigma_D = -\frac{e^2}{\hbar}\frac{1}{24\pi}\frac{\ln(\kappa d)}{\kappa d}\frac{1}{T\tau_{12}}. \tag{3a}$$

The appearance of a contribution which has a singular temperature dependence is not entirely unexpected for a diffusive system [14]. What is less common is the fact that the divergence is so pronounced, $\sim T^{-1}$, instead of being logarithmic, as in the case of Altshuler–Aronov interaction corrections to the conductivity of 2D systems [14]. The temperature dependence becomes even more singular at smaller temperatures, $T < (\kappa d)\hbar/\tau_{12}$ [13]

$$\sigma_D = -\frac{e^2}{\hbar}\frac{3\zeta(3)}{8\pi^4}\frac{\ln(T\tau_{12})}{(T\tau_{12})^2}. \tag{3b}$$

The temperature divergences in Eqs. (3a) and (3b) are cut off by the system size, L, at the Thouless temperature, $\hbar D/L^2$. For systems larger than the mean propagation distance within one layer, $\sqrt{D\tau_{12}}$, there is an additional temperature range $\hbar D/L^2 < T < \hbar/\tau_{12}$. Here the exchange contribution to the transconductance is dominated by multiple tunneling processes, leading to the form

$$\sigma_D = -\frac{e^2}{\hbar}\frac{1}{8\pi^2}\ln(T\tau_{12})^{-1}. \tag{3c}$$

This expression differs from the Altshuler–Aronov interaction correction to the 2D conductivity [14] only by a factor of 1/4, which reflects the peculiarity of the drag measurement setup. Where the current flows only in one half of the system and the potential is measured in the other half.

We now turn to the derivation of Eqs. (3a)–(3c). The Hamiltonian of the system is given by

$$H = H_1 + H_2 + H_{\mathrm{int}} + H_T, \tag{4}$$

where $H_{1(2)}$ is the Hamiltonian of the first (second) isolated layer, including elastic disorder, and H_{int} includes interlayer as well as intralayer Coulomb interactions. The first three terms on the r.h.s. of Eq. (4) are traditionally involved in the description of the drag effect [8–10]. We add the term describing pointlike tunneling processes

$$H_T = V\sum_{i=1}^{N}\sum_{k,p} e^{ir_i(k-p)}a_k^\dagger b_p + \mathrm{h.c.}, \tag{5}$$

where $a(a^\dagger)$ and $b(b^\dagger)$ are the annihilation (creation) operators of electrons in the first and the second layers respectively, and r_i, $i = 1\ldots N$, are the random positions

of N bridges. The tunneling amplitude V is related to the lifetime τ_{12} as $2\pi\nu L^2 N|V|^2 = \hbar/\tau_{12}$ (cf. Eq. (2)).

FIG. 1 Two diagrams contributing to the transconductance that are second order in tunneling (denoted by $\times$). Full lines with arrows are electron Green functions, dashed lines represent diffusons, and wavy lines screened interactions. Two additional diagrams with arrows in the opposite direction should be included. The numbers indicate the layer index.

At various temperatures the exchange contribution to the transconductance originates from the different processes. For example, for $(\kappa d)\hbar/\tau_{12} < T < \hbar/\tau$ the dominant contribution is second order in tunneling and is given by the four diffuson diagrams depicted in Fig. 1. Performing an analytical continuation and taking the dc limit in a standard manner [15], one finds for the exchange contribution to the transconductance

$$\sigma_D = i\frac{\sigma}{4\pi}\int_{-\infty}^{\infty}d\omega\frac{\partial}{\partial\omega}\left[\omega\coth\frac{\omega}{2T}\right]F_{xx}(\omega). \tag{6}$$

A straightforward summation over the fast electronic momenta leads to $F_{xx}(\omega) = F_{xx}^{(\mathrm{a})}(\omega)$ with

$$F_{xx}^{(\mathrm{a})}(\omega) = \frac{8}{\tau_{12}}\sum_{Q}DQ_x^2\frac{U_{12}(Q,\omega)}{(DQ^2 - i\omega)^4}, \tag{7}$$

where $U_{12}(Q,\omega)$ is the dynamically screened interlayer Coulomb interaction. In the diffusive limit ($\omega < \hbar/\tau$) it was found in Refs. [8,9] (see also Eqs. (13) and (14) below). Substituting Eq. (7) into Eq. (6) and carefully performing frequency and slow momenta integrations, one recovers Eq. (3a).

At somewhat smaller temperatures the fourth order (in the tunneling amplitude) processes may become dominant. Although being smaller by the additional factor of $|V|^2 \propto 1/\tau_{12}$, it contains intra– (instead of inter–) layer interaction, which is stronger by a $\kappa d > 1$ factor. Moreover, the fourth order term has a more singular temperature dependence (because additional diffusons poles are involved) which makes it eventually dominant at $T < (\kappa d)\hbar/\tau_{12}$. Examples of the diagrams, contributing to the fourth order are depicted in Fig. 2. The calculation of these diagrams follows the steps outlined above and leads to the result given in Eq. (3b). Instead of reproducing these calculations we develop a general formalism suitable for multi-tunneling processes, and obtain the fourth order contribution as a particular case.
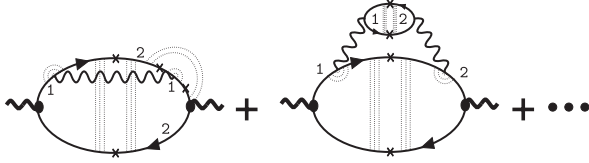
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FIG. 2 Examples of diagrams contributing to the transconductance that are fourth order in tunneling. Diagrams with interaction lines connecting "upper" and "lower" Green functions, as well as diagrams with an opposite direction of electron lines should also be included. The numbers indicate the layer index.

The treatment of multiple tunneling processes through randomly placed bridges is similar to the theory of metals with spin–flipping impurities [16]. In our case the role of the spin z component is played by the layer index. The main difference is that the different "spin" components are subjected to different (and uncorrelated) impurity potentials. We introduce the matrix notations for the diffusons $D_{mn}^{kl}(Q,\omega)$, where the indices $k,l,m,n = 1,2$ indicate the layer index for each of the four incoming and outgoing legs. Such a matrix diffuson is a solution of the following Bette–Salpeter equation

$$D_{mn}^{kl} = W_{mn}^{kl} + \sum_{i,j=1}^{2} W_{mj}^{ki} \zeta(Q,\omega)D_{jn}^{il}, \qquad (8)$$

where $\zeta(Q,\omega) = \tau^{-1} + \tau_{12}^{-1} - (DQ^2 - i\omega)$, and the combined tunneling and impurity scattering amplitude is given by

$$W_{mn}^{kl} = \frac{1}{\tau}\delta_{kl}\delta_{mn}\delta_{km} + \frac{1}{\tau_{12}}(1 - \delta_{kl})(1 - \delta_{mn}). \qquad (9)$$

Since the elastic disorder in the two layers is uncorrelated one may look for a solution of Eq. (8) in the form

$$D_{mn}^{kl}(Q,\omega) = A\delta_{kl}\delta_{mn}\delta_{km} + B(1 - \delta_{kl})(1 - \delta_{mn}), \quad (10)$$

then in the limit $\tau_{12} \gg \tau$ one finds

$$A, B = \frac{1}{2}\left[\frac{1}{DQ^2 - i\omega} \pm \frac{1}{DQ^2 - i\omega + 2/\tau_{12}}\right]. \qquad (11)$$

The function $F_{xx}(\omega)$ which enters in expression (6) for the transconductance may be written as (cf. the diagrams in Fig. 1)

$$F_{xx}(\omega) = 4\sum_{Q} DQ_x^2 \sum_{knlm} U_{kn}D_{lk}^{1k}\left[D_{12}^{lm} + D_{1m}^{l2}\right]D_{2n}^{mn}. \qquad (12)$$

Here $U_{kn}(Q,\omega)$ is the dynamically screened interaction between layers k and n. In the RPA approximation it is given by a solution of

$$U_{kn} = U_{kn}^0 - \sum_{l,m=1}^{2} U_{kl}^0 \Pi_{lm} U_{mn}, \qquad (13)$$

where U_{kn}^0 is the matrix of bare interactions [6–10] and $\Pi_{kn}(Q,\omega)$ is the polarization matrix which in the present case takes the form

$$\Pi_{kn}(Q,\omega) = \nu\left[\delta_{kn} + i\omega D_{kn}^{kn}(Q,\omega)\right]. \qquad (14)$$

Note, that by virtue of Eqs. (10) and (11) Π_{kn} satisfy the particle conservation law: $\sum_n \Pi_{kn}(Q = 0, \omega) = 0$.

Now, we are in the position to calculate $F_{xx}(\omega)$ for various values of ω. The second order contribution to the exchange transconductance (Fig. 1) comes from the expression (cf. Eq. (12)) $U_{12}D_{11}^{11}D_{12}^{12}D_{22}^{22} = U_{12}A^2B$. Expanding now up to the lowest order in $\hbar/\tau_{12}$, one obtains Eq. (7). The fourth order originates from terms like $U_{11}D_{11}^{11}D_{12}^{12}D_{21}^{21} = U_{11}AB^2$, which should be taken in the lowest non-vanishing order in $\hbar/\tau_{12}$, and from the term $U_{12}D_{11}^{11}D_{12}^{12}D_{22}^{22}$ where $U_{12} = -U_{11}\Pi_{12}U_{22}$. As a result one obtains $F_{xx}(\omega) = F_{xx}^{(b)}(\omega)$ with

$$F_{xx}^{(b)}(\omega) = \frac{8}{\tau_{12}^2}\sum_{Q}DQ_x^2\left[\frac{U_{11} + U_{22}}{(DQ^2 - i\omega)^5} - \frac{U_{11}\Pi_{12}U_{22}\tau_{12}}{(DQ^2 - i\omega)^4}\right]. \qquad (15)$$

The two terms on the r.h.s. of Eq. (15) are represented by the two classes of diagrams exemplified in Fig. 2. Substitution of Eq. (15) in Eq. (6) and integration over frequency and slow momenta yields Eq. (3b). It is easy to see that this contribution is the dominant one at $T < (\kappa d)\hbar/\tau_{12}$.

Finally in the limit of $\omega < \hbar/\tau_{12}$ one may neglect the second term in the expression for A and B [Eq. (11)] and then $D_{11}^{11} = D_{22}^{22} = D_{12}^{12} = D_{21}^{21} = (1/2)\left(DQ^2 - i\omega\right)^{-1}$. This yields for $\omega < 1/\tau_{12}$, $F_{xx}(\omega) = F_{xx}^{(c)}(\omega)$ with

$$F_{xx}^{(c)}(\omega) = \sum_{Q}DQ_x^2\frac{U_{11} + 2U_{12} + U_{22}}{(DQ^2 - i\omega)^3}. \qquad (16)$$

Eqs. (13) and (14) in the present case lead to $U_{11} + U_{12} = U_{12} + U_{22} = \nu^{-1}(DQ^2 - i\omega)/DQ^2$. As a result of slow momenta summation one obtains for $\omega < \hbar/\tau_{12}$: $F_{xx}^{(c)}(\omega) = ie^2/(4\pi\sigma\omega)$. After substitution of $F_{xx}^{(c)}(\omega)$ into Eq. (6) and integrating over the frequency one obtains a logarithm [Eq. (3c)], similar to the logarithm in Ref. [14]. For finite size systems the slow momentum integration should be replaced by a discrete summation. This results in placing the low–frequency cutoff, $\omega \sim \hbar D/L^2$, in Eqs. (7), (15) and (16). As a consequence the temperature dependence of Eqs. (3a)–(3c) is flattened out at $T < \hbar D/L^2$. In order to find the exact exchange contribution to transconductance in the transitions between the various regimes (including the temperature independent parts) one may directly substitute Eq. (12) into Eq. (6) and evaluate the resulting double integral.

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Finally we briefly discuss the ballistic limit, $\hbar/\tau < T$. For such a temperature range one should consider the diagrams depicted in Fig. 1 without the diffusons. In this case the function $F_{xx} = F_{xx}^{(\text{bal})}$ is given by

$$F_{xx}^{\text{bal}}(\omega) = \frac{4\tau^2}{D\tau_{12}} \sum_Q U_{12}(Q) \left[\frac{Q_x}{Q^2} \left(1 - \frac{1}{\sqrt{1 + X^2}} \right) \right]^2 ,$$

(17)

where $X \equiv v_F Q/(-i\omega + 1/\tau)$, and $U_{12}(Q)$ is the screened interlayer interaction in the clean limit [6–10]. The slow momenta summation and frequency integration according to Eq. (6) lead to a negative and temperature independent transconductance, which is given by:

$$\sigma_D = -\frac{e^2}{\hbar} \frac{\pi}{32} \frac{1}{\kappa d} \frac{v_F \tau^2}{d\tau_{12}} .$$

(18)

One should not overestimate the significance of this negative constant exchange contribution because of the existence of two additional "parasitic" effects. Both contribute positive temperature independent constants to the measured transconductance.

The first effect is related to a possibility of momentum conserving tunneling. In this case the transconductance is nonzero (and positive) at zero temperature even *without* any *e–e* interactions. It originates from the delocalization of the wavefunctions between the two layers, and may be described in the framework of the single–particle picture [17,18]. This type of (tunneling) transconductance may be differentiated from other mechanisms by a strong sensitivity to an *in-plane* magnetic field or a gate voltage. A Fermi–surface mismatch introduced by these factors leads to a rapid suppression of the transconductance [17,18]. The second effect originates from the classical (Hartree) interactions between electrons. In the presence of many bridges it may be visualized by a network of classical resistors. A simple calculation based on Kirchoff's laws leads to a positive transconductance. For $L > \sqrt{D\tau_{12}}$ the network model predicts that approximately half of the current applied to the first layer eventually leaks out to the second one. In this case the transresistance practically reduces to the resistance of a single layer of a doubled width [thus it is not surprising that we have recovered the Altshuler–Aronov interaction correction at low temperatures and at a large system size, Eq. (3c)]. However, we stress that both mechanisms mentioned above lead to a temperature independent effect, which is easily distinguishable from the strong temperature dependent exchange contribution to the transconductance.

To conclude we have examined the influence of point-like tunneling between the two layers, in combination with *e-e* interactions, on the drag effect and the transconductance. We found a distinguishable exchange contribution to the transconductance, which is negative, has a strong temperature dependence and is non-vanishing at zero temperature. The magnitude of the effect under optimal conditions may reach a fraction of $e^2/2\pi\hbar$ (for the transconductance).

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